Polynomial representation of DNA and Proteins

– Hetero number theory –

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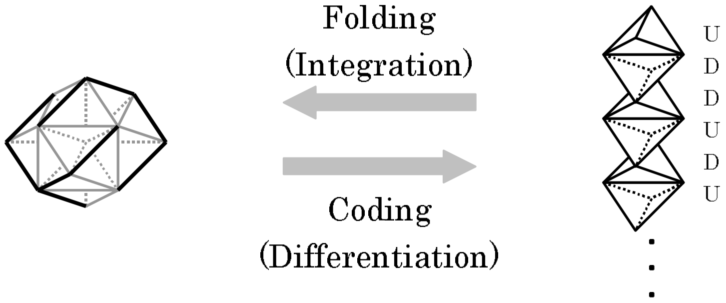
0. Aim of this talk

(1)Analysis of Hetero numbers ($\mathbb{HN}$)



Folding
(Integration)

Coding
(Differentiation)

$$D^2[xyw+xz+yz+wz] = U\text{-}D\text{-}D\text{-}U\text{-}D\text{-}U\text{-}U\text{-}D\text{-}U\text{-}D\text{-}D\text{-}U\text{-}D\text{-}U\text{-}U\text{-}D\text{-}U\text{-}D\text{-}D\text{-}U\text{-}D\text{-}U\text{-}U\text{-}D$$

D^2[protein] ⇔ Gene

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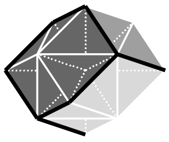
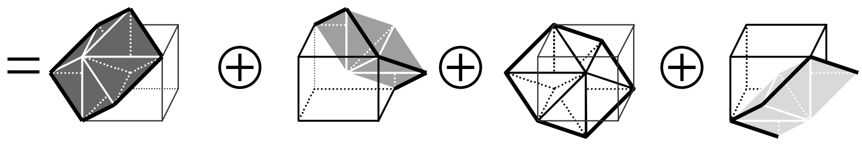
0. Aim of this talk

(2)Algebra of $\mathbb{HN}$



Protein-protein interaction
⇔ addition



$$xyz+xyw+xzw+yzw$$
$$= (xyz+xyw+xzw)\oplus(xyz+xyw+yzw)\oplus(xyz+xzw+yzw)\oplus(xyw+xzw+yzw)$$

Protein ⇔ Number

1.0 Other studies

(1)

1.0 Other studies

(2)

1.1 Motivation

(1) Features of hetero numbers ($\mathbb{HN}$) - la raison d'etre -

(a) **Biology** and $\mathbb{HN}$:

- 2^{nd} derivative ⇔ Genetic coding
- Integration ⇔ Protein folding
- Addition ⇔ Protein-protein interaction

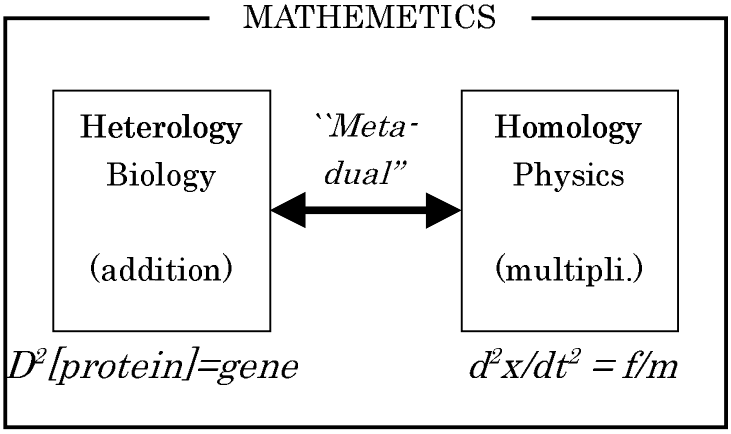
(b) **Natural numbers** and $\mathbb{HN}$:

- Higher dimensional extension of $\mathbb{N}$ w.r.t. addition.
- Genetic coding of $\mathbb{N}$ (2^{nd} derivative of $\mathbb{N}$)

$\mathbb{HN}$ = A system of units for measuring shapes

1.1 Motivation

(2) Implicit paradigm - hetero. VS. homo. -



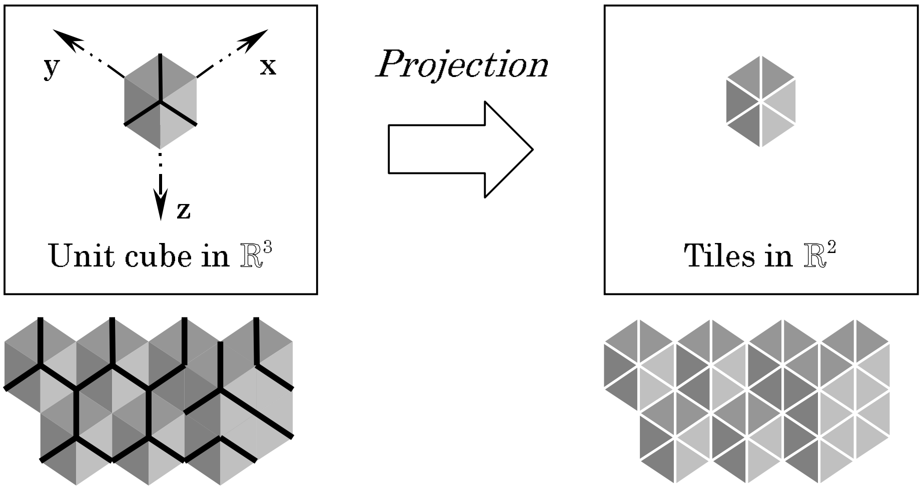
$D^2[protein]=gene$

$d^2x/dt^2 = f/m$

Heterology VS. Homology

1.2 Basic idea (1): peaks & valleys

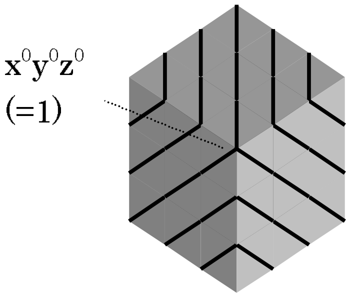
(1) Unit cube in $\mathbb{R}^3$



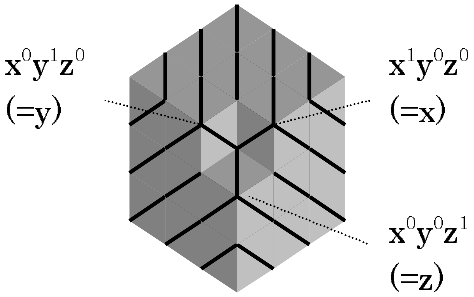
Unit cubes and Triangle tiles

1.2 Basic idea (1): peaks & valleys

(2) Drawings by patterns on cubes



One peak 1 of height 0



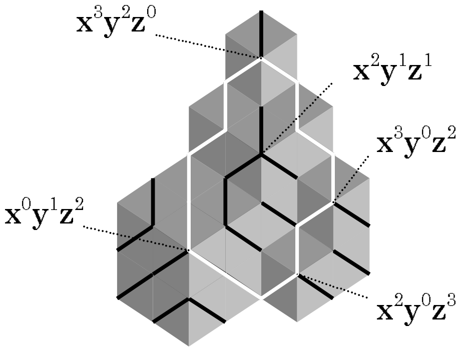
Three peaks x, y and z of height –1

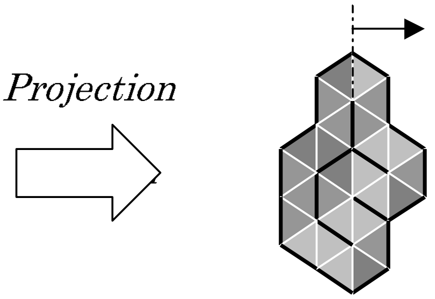
∗Notation: $x^l y^m z^n \Leftrightarrow (l,m,n)$
∗Height : $h(x^l y^m z^n) := -(l+m+n)$

Peaks and valleys

1.3 Basic idea (2)： analysis

(1) What is a hetero number ?





Peaks and valleys of f0 ***Closed orbit of f0***

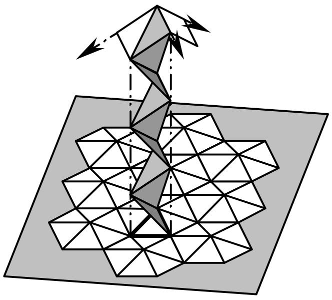
$f0 = x^3y^2+x^3z^2+x^2z^3+yz^2+x^2yz$ (5 peaks)

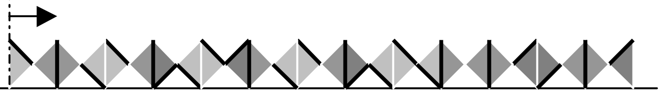
A hetero number ⇔ A set of closed orbits

1.3 Basic idea (2)： analysis

(2) Differential structure of$\mathbb{HN}$

Function ⇔ height
1st Derivative ⇔ height (mod 3)
2nd Derivative ⇔ up/down



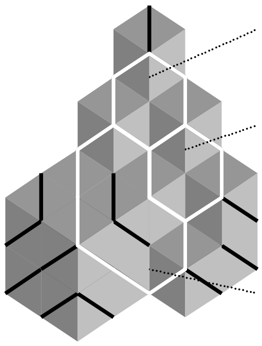


Height×(-1)	5 5 4 4 5 5 5 5 4 4 4 4 5 5 5 5 4 3 3 4 5 5 4 4 5 5
1st Derivative	1 1 2 2 1 1 1 1 2 2 2 2 1 1 1 1 2 0 0 2 1 1 2 2 1 1
2nd Derivative	DUUD DUDU UDUD DUDU UUDD DUUD DU

2nd Derivative = *Up* and *down* along an orbit

1.4 Basic idea (3) :　algebra

(1) Addittion of $\mathbb{HN}$



Orbit1: $f1=x^3y^2+x^3yz+x^2y^2z$

Orbit2: $f2= x^3z^2+x^3yz+x^2yz^2$

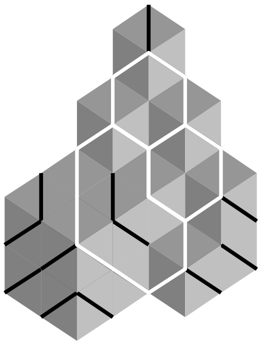
Orbit3: $f3= x^2z^3+yz^2+x^2y^2z$

$f = x^3y^2+x^3z^2+x^2z^3+yz^2+x^3yz+x^2y^2z$　*(6 peaks)*

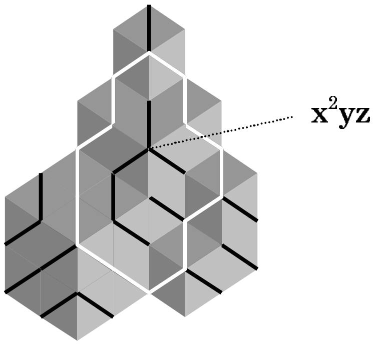
$$f = f1 \oplus f2 \oplus f3$$

1.4 Basic idea (3) :　algebra

(2) Action by monomial on $\mathbb{HN}$



$f= f1 \oplus f2 \oplus f3$



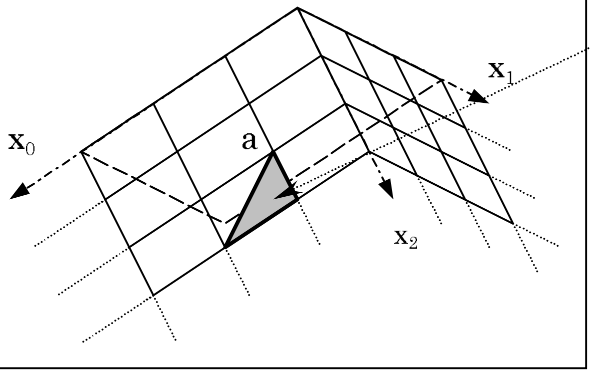
$f0$

$$f0 = (f1 \oplus f2 \oplus f3) * x^2yz$$

2.1 Set of polyhedrons

(1) Lattice and N-hedrons

$$\mathbf{L} := \{\, \mathbf{x}_0^{l0}\mathbf{x}_1^{l1} \ldots\ \mathbf{x}_{N-1}^{lN-1} : l_i \in \mathbb{Z} \,\} \sim \mathbb{Z}^N$$



3-hedron (Convex hull)

$*[a,g]=[\ a,\ ax_2,\ ax_2x_0]$

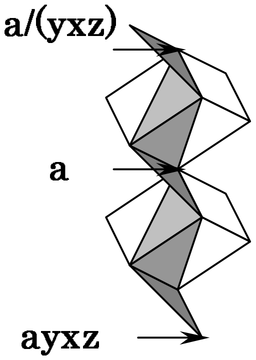
where $g=\begin{pmatrix}0&1&2\\2&0&1\end{pmatrix}$

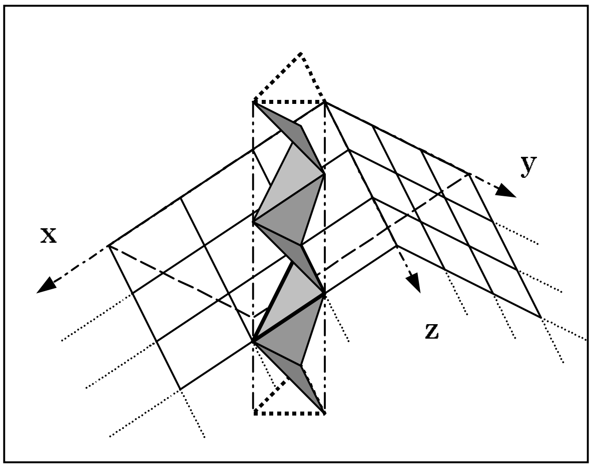
$= x_2x_0x_1$

$$[\mathbf{a},\mathbf{g}]_T := [\ \mathbf{a},\ \mathbf{a}\mathbf{x}_{g(0)},\ \ldots,\ \mathbf{a}\mathbf{x}_{g(0)}\mathbf{x}_{g(1)} \ldots \mathbf{x}_{g(N-2)}\ ] \quad (\,(a,g) \in L \times S_N\,)$$

2.1 Set of polyhedrons

(2) Set of N-hedrons





$[a/(yxz)\ ,\ zxy]_T$

$[a/(yx)\ ,\ xyz]_T$

$[a/y\ ,\ yzx]_T$

$[a\ ,\ zxy]_T$

$[az\ ,\ xyz]_T$

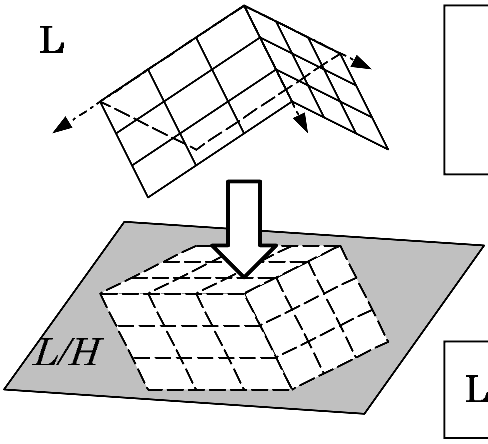
$[azx\ ,\ yxz]_T$

$$\mathbf{S}^{N-1} := \quad \{\ [\mathbf{a},\mathbf{g}]_T : (\mathbf{a},\mathbf{g}) \in \mathbf{L}\times S_N \}$$

* [Notation] $x_ix_jx_k \Leftrightarrow \begin{pmatrix}012\\ i\ j\ k\end{pmatrix}$

2.2 Projection π

(1) Projection of lattice



$L := \{ x_0^{l0} x_1^{l1} \ldots x_{N-1}^{lN-1} : l_i \in \mathbb{Z} \}$

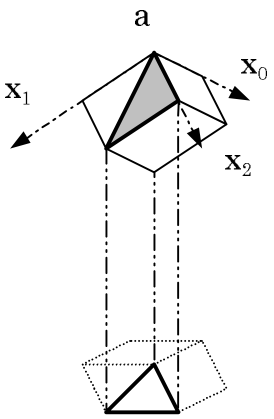
$H := \{ (x_0 x_1 \ldots x_{N-1})^{k} : l_i \in \mathbb{Z} \}$

$L/H \sim \{\mathbf{x}_0^{l0} \mathbf{x}_1^{l1} \ldots \mathbf{x}_{N-1}^{lN-1} : 0 \leq \sum l_i < N\}$

$$\pi : L \rightarrow L/H$$

2.2 Projection π

(2) Projection of N-hedrons



$[\mathbf{a},\mathbf{g'}]_T$ *Slant tile*

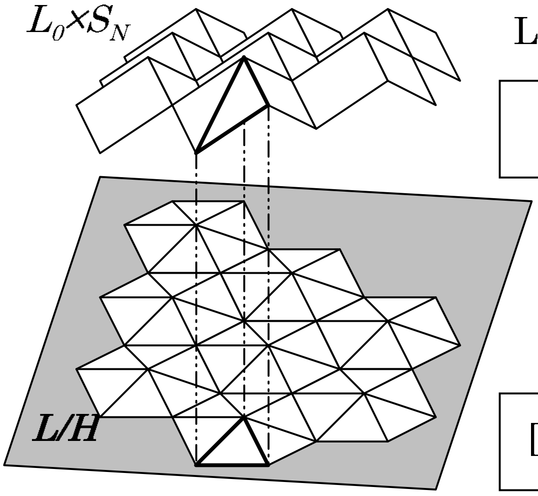
⇩

$[\pi(\mathbf{b}),\mathbf{g}]_B$ *Base tile*

$$[\pi(\mathbf{b}),\mathbf{g}]_B := [\pi(\mathbf{b}), \pi(\mathbf{b}\mathbf{x}_{g(0)}), \ldots, \pi(\mathbf{b}\mathbf{x}_{g(0)}\mathbf{x}_{g(1)} \ldots \mathbf{x}_{g(N-2)})] \subset \mathbb{R}^{N-1}$$

2.3 Base tiles

(1) N-hedron tiling of $\mathbb{R}^{N\text{-}1}$



$L_0 := \{ x_0^{l0} \ldots x_{N-1}^{lN-1} : l_i \in \mathbb{Z} , \sum l_i = 0 \}$

$[b,g]_T = [\ b,\ bx_{g(0)},\ bx_{g(0)}x_{g(1)}\ ]$

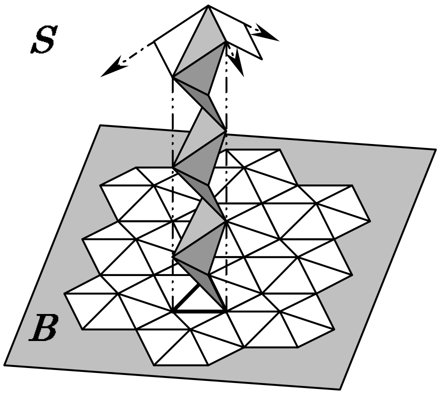
⇕

$[b,g]_B = [\ b,\ \pi(bx_{g(0)}),\ \pi(bx_{g(0)}x_{g(1)})\ ]$

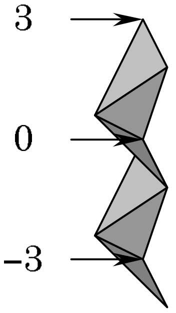
$$\mathbf{B}^{N\text{-}1} := \{ [\mathbf{b},\mathbf{g}]_B : (\mathbf{b},\mathbf{g}) \in \mathbf{L}_0 \times S_N \} \sim \mathbf{L}_0 \times S_N$$

2.3 Base tiles

(2) $\mathbb{Z}$ bundle over B



Height of slant tiles



$3 \Leftrightarrow [1/(yxz) , zxy]_T$
$2 \Leftrightarrow [1/(yx) , xyz]_T$
$1 \Leftrightarrow [1/y , yzx]_T$
$0 \Leftrightarrow [1 , zxy]_T$
$-1 \Leftrightarrow [z , xyz]_T$
$-2 \Leftrightarrow [zx , yxz]_T$

- $[x^l y^m z^n, g]_T \sim (\pi([x^l y^m z^n, g]_T) , -(\mathbf{l}+\mathbf{m}+\mathbf{n}))$

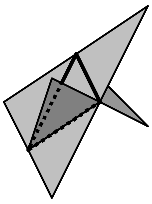
$$\mathbf{S}^{N-1} \sim \mathbf{B}^{N-1} \times \mathbb{Z}$$

2.4 Topology

(1) Topology of S

For N–2 faces of $[a,g]_T$

$[\ a_0,\ a^{(1)}_1,\ \ldots,\ a^{(1)}_{N-2},\ a_{N-1}\ ]$
$[\ a_0,\ a^{(2)}_1,\ \ldots,\ a^{(2)}_{N-2},\ a_{N-1}\ ]$
…
$[\ a_0,\ a^{(N-2)}_1,\ \ldots,\ a^{(N-2)}_{N-2},\ a_{N-1}\ ]$



For face $[\ a_0,\ \ldots,\ a_{N-2}\ ]$

$[\ a'_{N-1},a_0,\ \ldots,\ a_{N-2}\ ]$

$[\ a_0,\ \ldots,a_{N-2},\ a'_{N-1}\Pi\mathbf{x}_i\ ]$

For face $[\ a_1,\ \ldots,\ a_{N-1}\ ]$

$[\ a'_0,\ a_1,\ \ldots,\ a_{N-1},\ ]$

$[\ a_1,\ \ldots,\ a_{N-1},\ a'_0\Pi\mathbf{x}_i\ ]$

$[a,g]_T = [\ a_0,\ a_1,\ \ldots,\ a_{N-2},\ a_{N-1}\ ]$

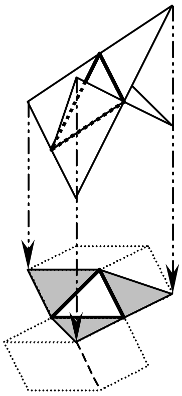
N+2 neighbors of a tile in S^{N-1}

2.4 Topology

(2) Topology of B

For N–2 faces of $[a,g]_T$

$\pi([\ a_0,\ a^{(1)}_1,\ \ldots,\ a^{(1)}_{N-2},\ a_{N-1}\ ])$
$\pi\ ([\ a_0,\ a^{(2)}_1,\ \ldots,\ a^{(2)}_{N-2},\ a_{N-1}\ ])$
…
$\pi\ ([a_0,a^{(N-2)}_1,\ldots,a^{(N-2)}_{N-2},a_{N-1}])$



For face $[\ a_0,\ \ldots,\ a_{N-2}\ ]$

$\pi\ ([\ a'_{N-1}\ ,a_0,\ \ldots,\ a_{N-2}\ ])$

For face $[\ a_1,\ \ldots,\ a_{N-1}\ ]$

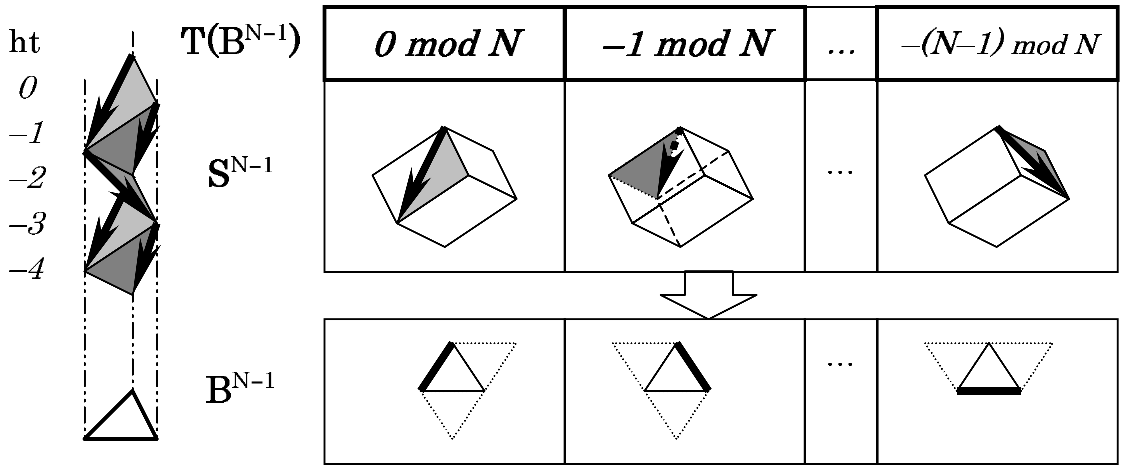
$\pi\ ([\ a'_0,\ a_1,\ \ldots,\ a_{N-1},\ ])$

$[b,g]_B = \pi([\ a_0,\ a_1,\ \ldots,\ a_{N-2},\ a_{N-1}\ ])$

N neighbors of a tile in B^{N-1}

2.5 Differential structure

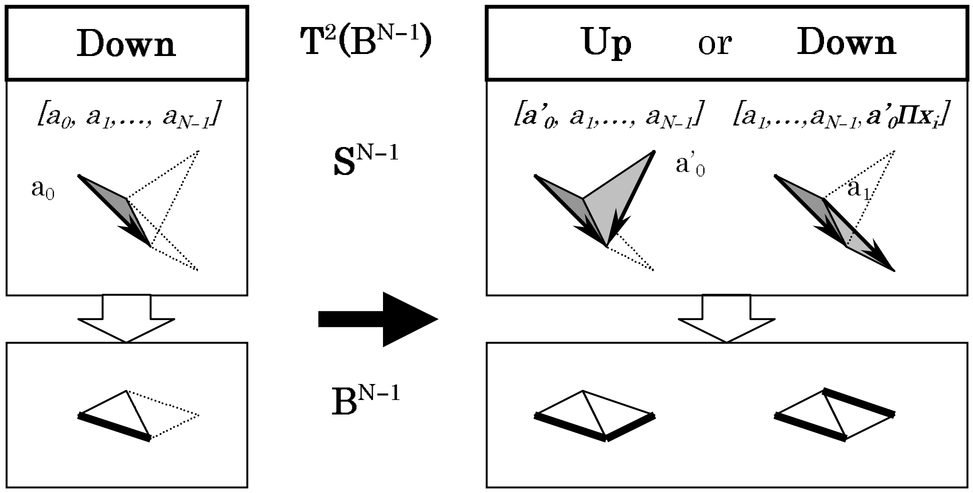
(1) 1st derivative $D[a,g]_T$ of $[a,g]_T$: **Gradient**



$$T(B^{N-1}) \sim B^{N-1} \times \mathbb{Z}/(N)$$

2.5 Differential structure

(2) 2nd derivative $D^2[a,g]_T$ of $[a,g]_T$: **Direction**

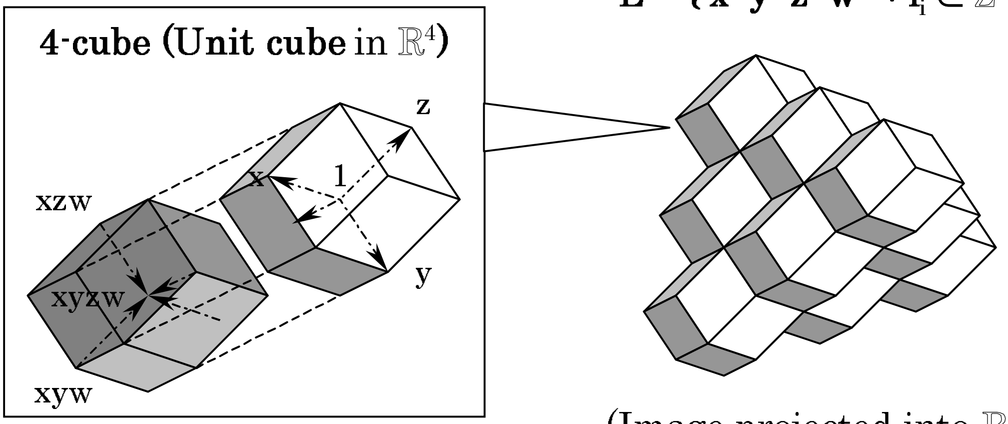


$$T^2(B^{N-1}) \sim B^{N-1} \times \{\text{Up, Down}\}$$

2.6 Example (N=4)

(1) 4-dimensional Lattice

$\mathbf{L} = \{ \mathbf{x}^{l0}\mathbf{y}^{l1}\mathbf{z}^{l2}\mathbf{w}^{l3} : l_i \in \mathbb{Z} \}$



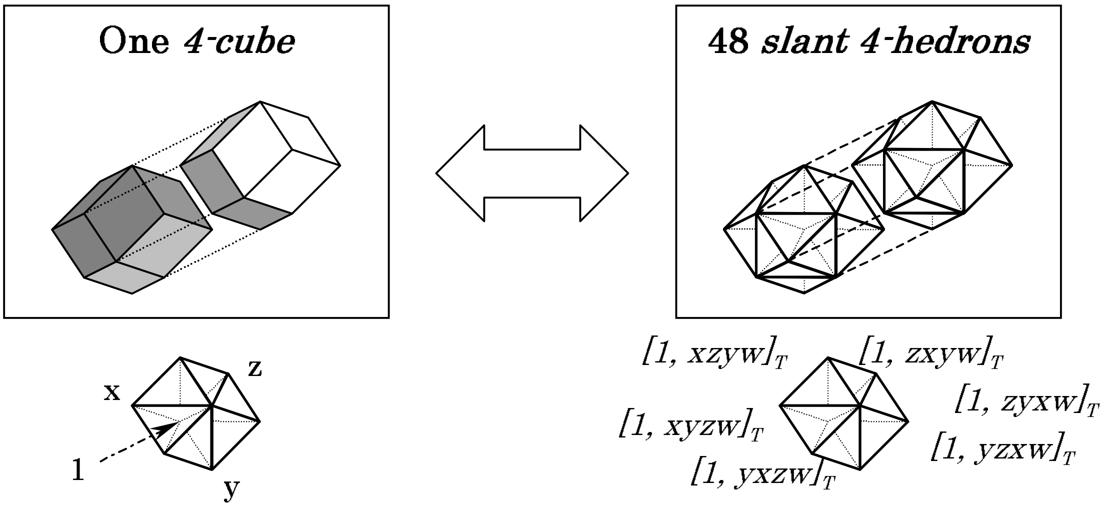
(Image projected into $\mathbb{R}^3$)

4-cube ⇔ Double covering of a dodecahedron

* 3-cube ⇔ Double covering of a *hexagon*

2.6 Example (N=4)

(2) The set of slant tiles

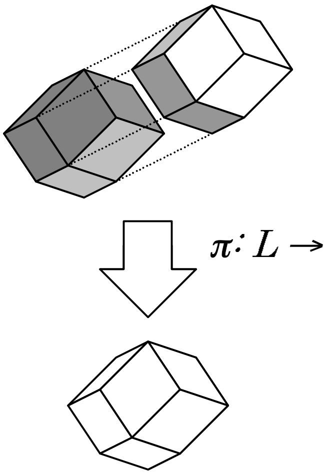


$$\mathbf{S}^{N-1} := \{ [\mathbf{a},\mathbf{g}]_T : (\mathbf{a},\mathbf{g}) \in \mathbf{L} \times \mathcal{S}_N \}$$

* One *3-cube* ⇔ 12 *slant tiles*

2.7 Example (N=4)

(1) Projection of lattice



π: L → L/H

$(l, m, n, k) \in \mathbb{Z}^4$

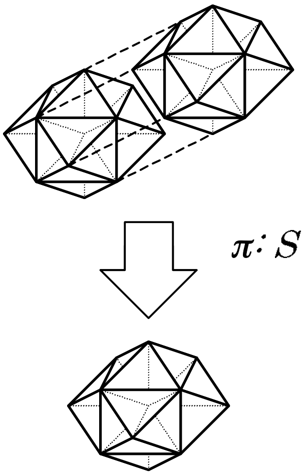
$(1,0,0,0) \rightarrow (1,0,0)$
$(0,1,0,0) \rightarrow (0,1,0)$
$(0,0,1,0) \rightarrow (0,0,1)$
$(0,0,0,1) \rightarrow (-1,-1,-1)$

$(l-k, m-k, n-k) \in \mathbb{Z}^3$

π : 4-cube → a rhombic dodecahedron

2.7 Example (N=4)

(2) Projection of 4-hedrons



π: S → B

48 slant 4-hedron tiles

(4-cube in $\mathbb{R}^4$)

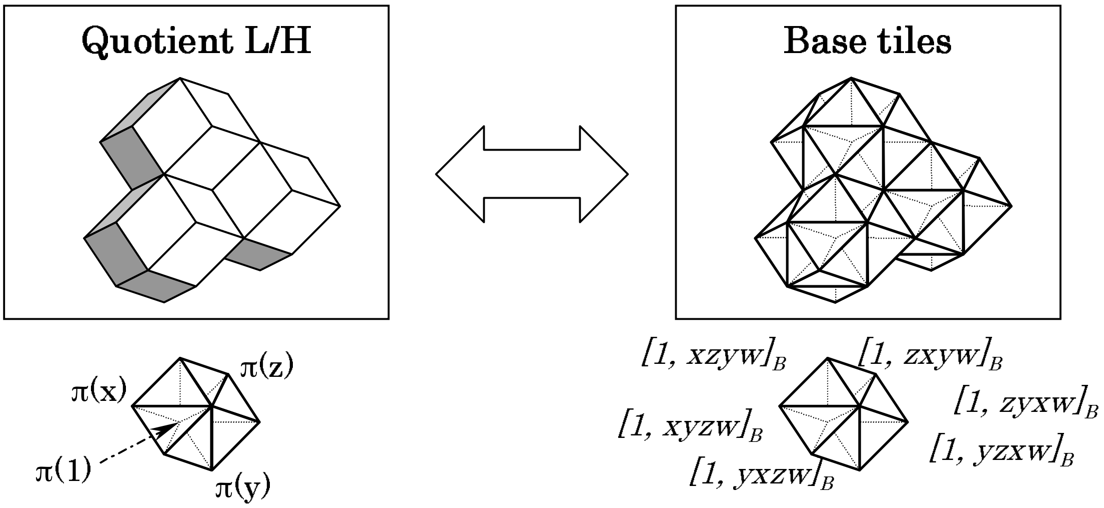
24 base 4-hedron tiles

(Rhombic dodecahedron in $\mathbb{R}^3$)

π : slant 4-hedron → base 4-hedron

2.8 Example (N=4)

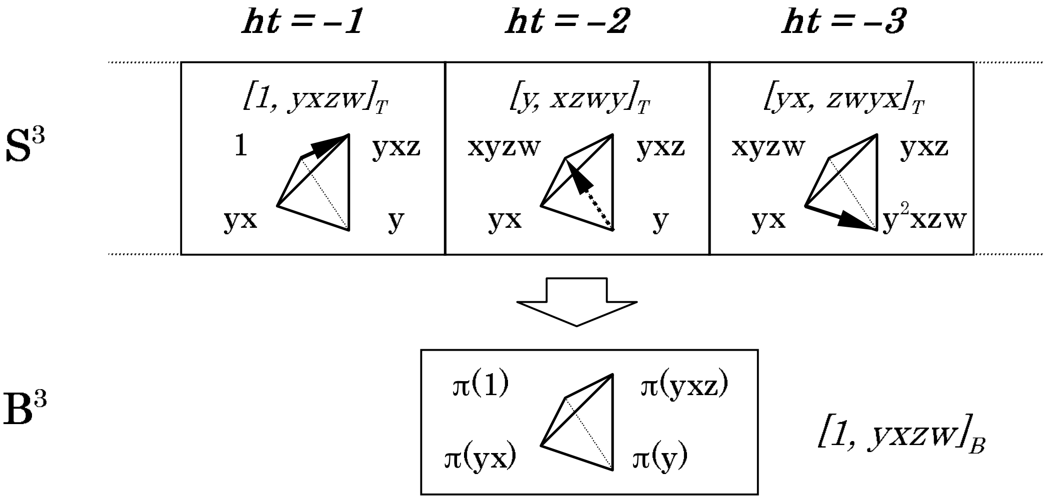
(1) 4-hedron tiling of $\mathbb{R}^3$



$$B^{N\text{-}1} := \{ [b,g]_B : (b,g)\in L_0\times \mathcal{S}_N \} \sim L_0 \times \mathcal{S}_N$$

2.8 Example (N=4)

(2) $\mathbb{Z}$ bundle over B

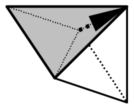


$$S^3 \sim B^3 \times \mathbb{Z}$$

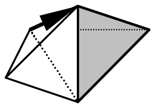
2.9 Example (N=4)

(1) Topology of S^3

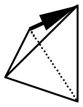
$[1, xyzw]_T$



$[1, yzxw]_T$

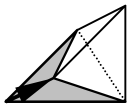


$[1, yxzw]_T$

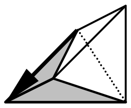


(ht= 0)

$[1/z, zyxw]_T$

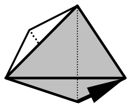


$[1, yxwz]_T$

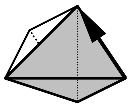


(ht= 1) (ht= 0)

$[y, xzyw]_T$



$[y/w, wxzy]_T$



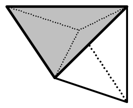
(ht = −1) (ht= 0)

6 neighbors of a slant 4-hedron tile

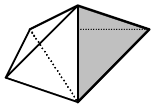
2.9 Example (N=4)

(2) Topology of B^3

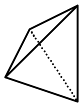
$[1, xyzw]_B$



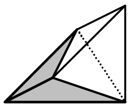
$[1, yzxw]_B$



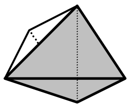
$[1, yxzw]_B$



$[1, yxwz]_B$



$[y/w, wxzy]_B$

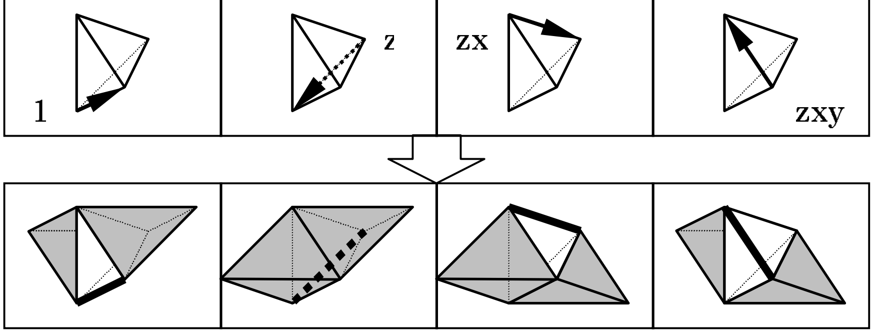


4 neighbors of a base 4-hedron tile

2.10 Example (N=4)

(1) 1st derivative : Gradient of slant tiles over $[1 ,zxyw]_B$

$T(B^3)$	*0 mod 4*	*–1 mod 4*	*–2 mod 4*	*–3 mod 4*
S^3	$[1 , zxyw]_T$	$[z , xywz]_T$	$[zx , ywzx]_T$	$[zxy, wzxy]_T$
B^3				

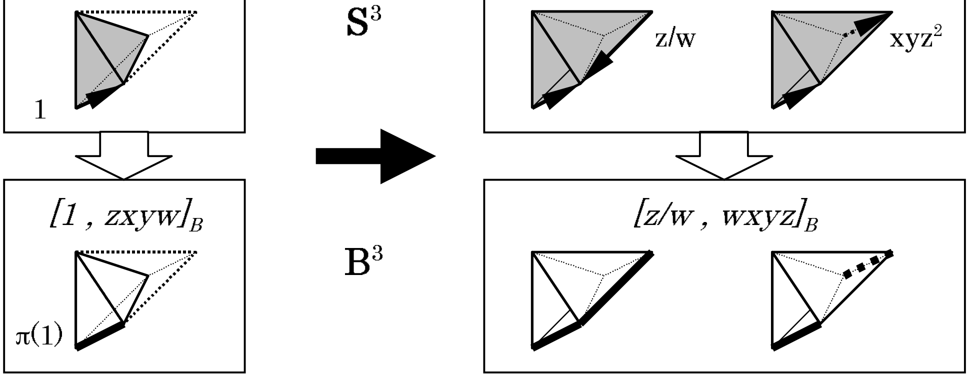


$$T(B^3) \sim B^3 \times \mathbb{Z}/(4)$$

2.10 Example (N=4)

(2) 2nd derivative : Direction of a local orbit at $[z/w , wxyz]_B$

Down	$T^2(B^3)$	Up or Down
$[1 , zxyw]_T$	S^3	$[z/w , wxyz]_T$ $[z , xyzw]_T$
$[1 , zxyw]_B$	B^3	$[z/w , wxyz]_B$



$$T^2(B^3) \sim B^3 \times \{Up, Down\}$$

3.1 Polynomial section of S

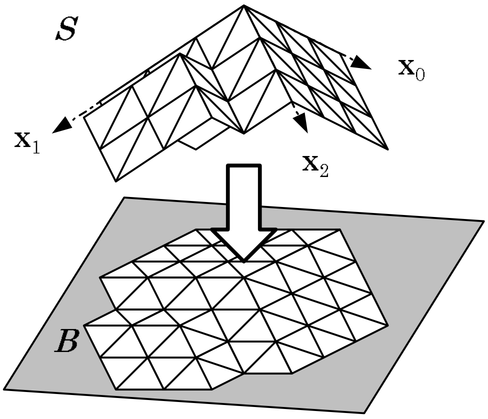
(1) Powerset of L

$f = a_0 + \ldots + a_m \in PL$

(Additive representation)

⇔

Sf := Slopes of peaks { a_i }

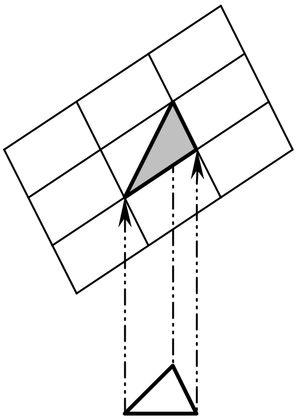


PL = The set of all finite subsets of L

3.1 Polynomial section of S

(2) Polynomial section of S

Slope Sf over $[b,g]_B$

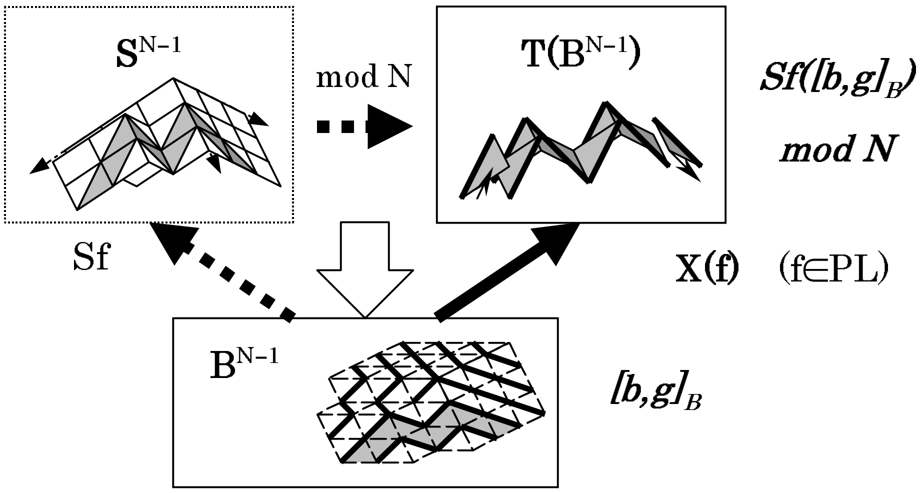


$$\begin{tikzcd} S^{N-1} & Sf([b,g]_B) \\ B^{N-1} & {[b,g]_B} \arrow[u, "Sf\ (f \in PL)"'] \end{tikzcd}$$

$Sf([b,g]_B) =$ the slope of Sf over $[b,g]_B$

3.2 Affine vector field

(1) Affine vector field X(f)



S^{N-1}

mod N

$T(B^{N-1})$

$Sf([b,g]_B)$

mod N

Sf

X(f) (f∈PL)

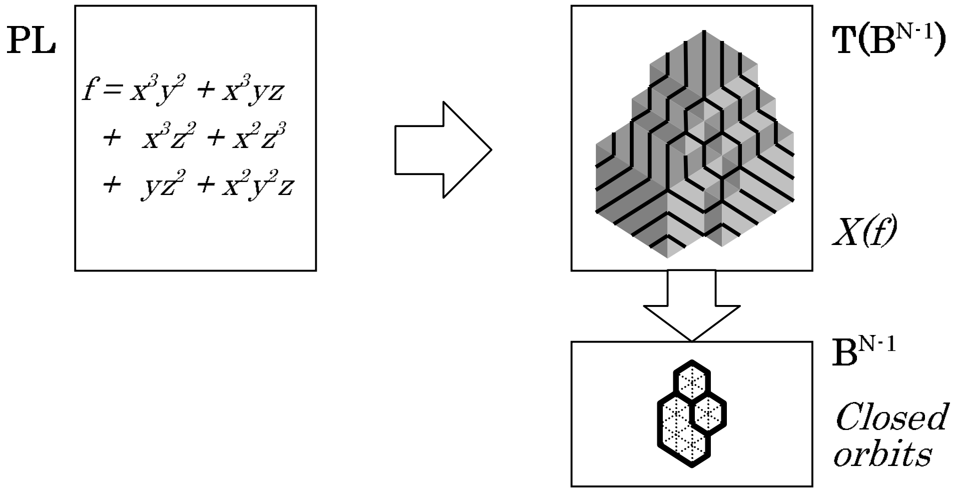
B^{N-1}

$[b,g]_B$

X(f) := Sf mod N

3.2 Affine vector field

(2) Affine loops



PL

$$f = x^3y^2 + x^3yz + x^3z^2 + x^2z^3 + yz^2 + x^2y^2z$$

$T(B^{N-1})$

X(f)

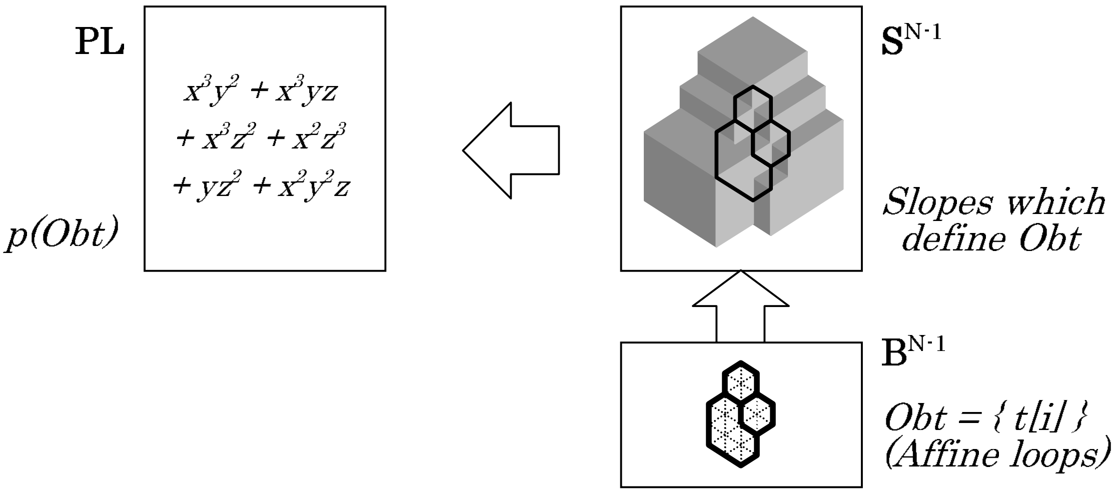
B^{N-1}

Closed orbits

ALP^{N-1} = { closed orbits of X(f) : f∈PL }

3.3 Polynomial representation

(1) Polynomial representation of affine loops



PL

$x^3y^2 + x^3yz$
$+ x^3z^2 + x^2z^3$
$+ yz^2 + x^2y^2z$

p(Obt)

S^{N-1}

Slopes which define Obt

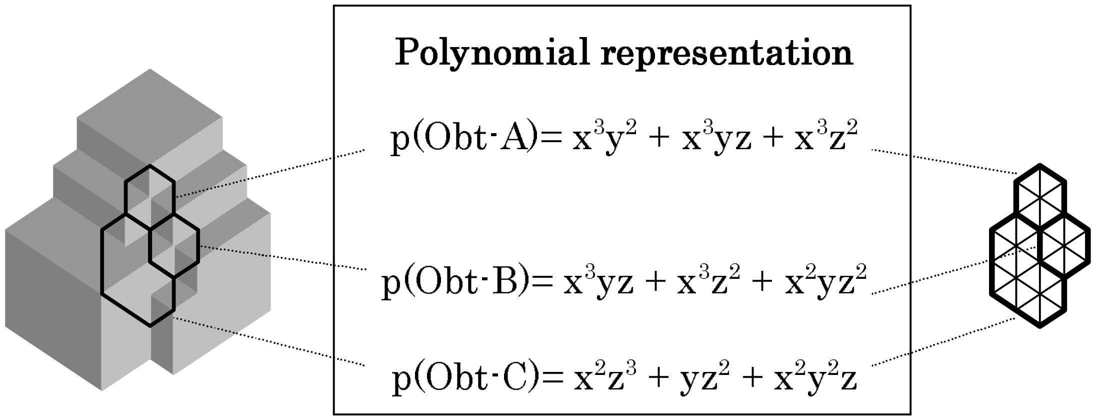
B^{N-1}

Obt = { t[i] }
(Affine loops)

p(Obt) := the peaks which define Obt

3.3 Polynomial representation

(2) Uniqueness of the reprensentation



Polynomial representation

$p(Obt\text{-}A)= x^3y^2 + x^3yz + x^3z^2$

$p(Obt\text{-}B)= x^3yz + x^3z^2 + x^2yz^2$

$p(Obt\text{-}C)= x^2z^3 + yz^2 + x^2y^2z$

$p(Obt) \equiv p(Obt') \bmod \prod x_i \quad \Leftrightarrow \quad Obt = Obt'$

4.1 Affine hetero numbers

(1) Definition

PL

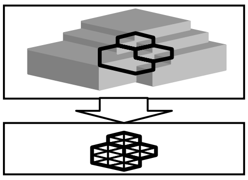
$p(V) := \sum\{p(Obt) : Obt \in V\}$

$$f = x^3y^2 + x^3yz + x^3z^2 + x^2z^3 + yz^2 + x^2y^2z$$

$\Longleftrightarrow$

ALP

$V(f) := \{Obt : p(Obt) \subset f\}$



$f=\sum a_i$ → $V(f)$

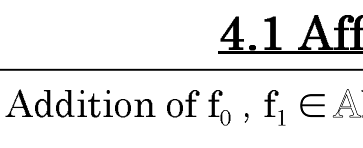
$p(V)=\sum\{p(Obt_i)\}$ ← $V=\{Obt_i\}$

$$\mathbb{AHN}^{N\text{-}1} = \{ f \in PL : p(V(f)) = f \}$$

4.1 Affine hetero numbers

(2) Addition of f_0 , $f_1 \in \mathbb{AHN}$

$$f_0 \oplus f_1 := f \quad \text{if} \quad V(f) = V(f_0)+V(f_1)$$



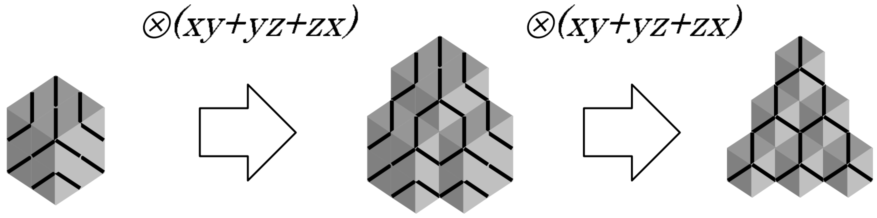
$f = p(Obt\text{-}A) \oplus p(Obt\text{-}B) \oplus p(Obt\text{-}C)$

$V(f)=Obt\text{-}A + Obt\text{-}B + Obt\text{-}C$

$$V(f_0 \oplus f_1) = V(f_0\ ) + V(f_1)$$

4.2 Algebra of affine hetero numbers

(1) Multiplication of f_0 , $f_1 \in$ AHN



1

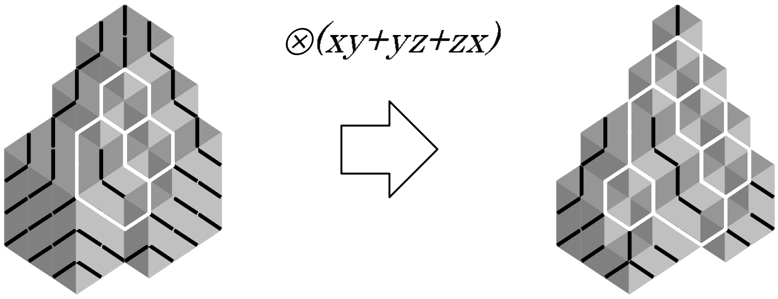
$1 \otimes (xy+yz+zx)$
$= xy+yz+zx$

$(xy+yz+zx) \otimes (xy+yz+zx)$
$= x^2y^2+xy^2z+y^2z^2+xyz^2$
$+x^2z^2+x^2yz$

$$f_0 \otimes f_1 := \sum\{ a_{0j}a_{1k} \} \quad (f_0 = \sum_k a_{0k},\ f_1 = \sum_k a_{1k})$$

4.2 Algebra of affine hetero numbers

(2) ⊕ and ⊗



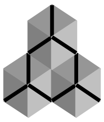
$(p(O_0) \oplus p(O_1) \oplus p(O_2))$

$p(O'_0) \oplus p(O'_1) \oplus p(O'_2) \oplus p(O'_3) \oplus p(O'_4)$

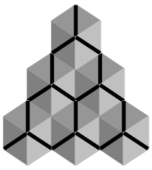
$$(\sum p(O_i)) \otimes (xy+yz+zx) = \sum p(O'_j)$$

4.3 Extension of natural numbers

(1) Extension w.r.t. addition



Δ(1)



Δ(2)

$\Delta : \mathbb{N} \rightarrow \mathbb{AHN}^{N-1}$, $\Delta(0) := 1$,

$$\Delta(n+1) := \Delta(n) \otimes (e/x_0 + \ldots + e/x_{n-1}).$$

* $e := x_0 x_1 \ldots x_{n-1}$

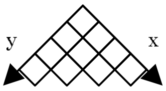
- $\Delta(m+n) = \Delta(m) \otimes \Delta(n)$
- $\Delta(n) = a_0 c \oplus a_1 c \oplus \ldots \oplus a_k c$, (*Sum of labeled units*)

  where $\sum a_i = \Delta(n-1)$ and $c = (e/x_0 + e/x_1 + \ldots + e/x_{n-1})$

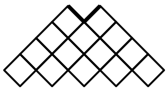
Higher dimensional natural numbers Δ(n)

4.3 Extension of natural numbers

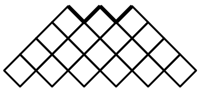
(2) Natural numbers as affine hetero number



Δ(0)



Δ(1)



Δ(2)

$\Delta : \mathbb{N} \rightarrow \mathbb{AHN}^{1}$, $\Delta(0) := 1$,

$$\Delta(n+1) = \Delta(n) \otimes (x+y).$$

- $\Delta(1) = x+y$
- $\Delta(2) = x(x+y) \oplus y(x+y) = x^2+xy+y^2$
- $\Delta(n) = x^{n-1}(x+y) \oplus x^{n-2}y(x+y) \oplus \ldots \oplus y^{n-1}(x+y)$

  $= \quad x^n+x^{n-1}y+x^{n-2}y+ \ldots +xy^{n-1}+y^n$

Natural numbers

5.1 Motivation

(1) Motivation

Analytic definition

AHN = Closed orbits

⇔

Algebraic definition

PHN=Clusters of AHN

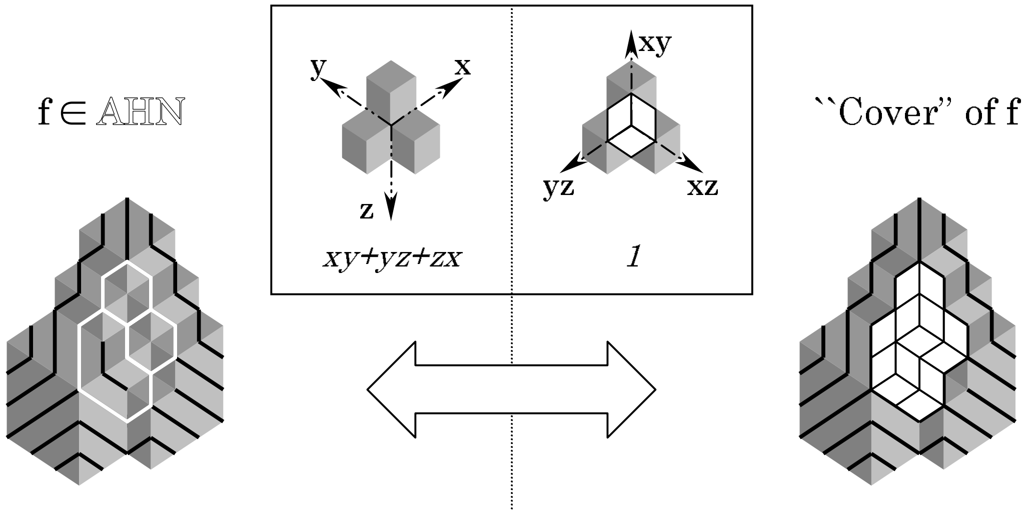
AHN=Factors of PHN

- Interaction among AHN ← PHN

Algebraic characterization ofAHN

5.1 Motivation

(2) Idea

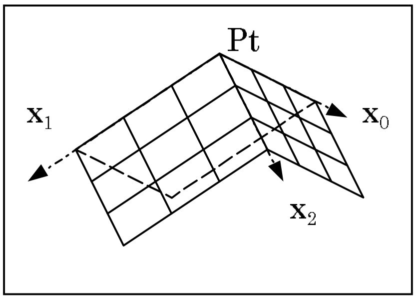


``Cover'' of an affine hetero number

5.2 Join operator $\vee$

(1) Two order relations in L

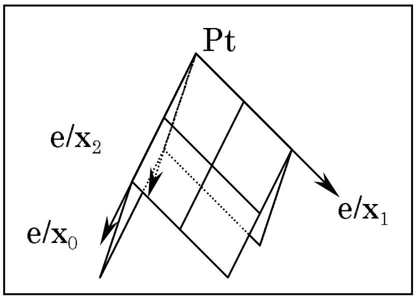
Area below Pt w.r.t. T



$\prod \mathbf{x}_i^{li} \geq_T \prod \mathbf{x}_i^{mi}$

$\Leftrightarrow l_i \leq m_i \leq (0 \quad i < N)$

Area below Pt w.r.t. C



$\prod (e/\mathbf{x}_i)^{li} \geq_C \prod (e/\mathbf{x}_i)^{mi}$

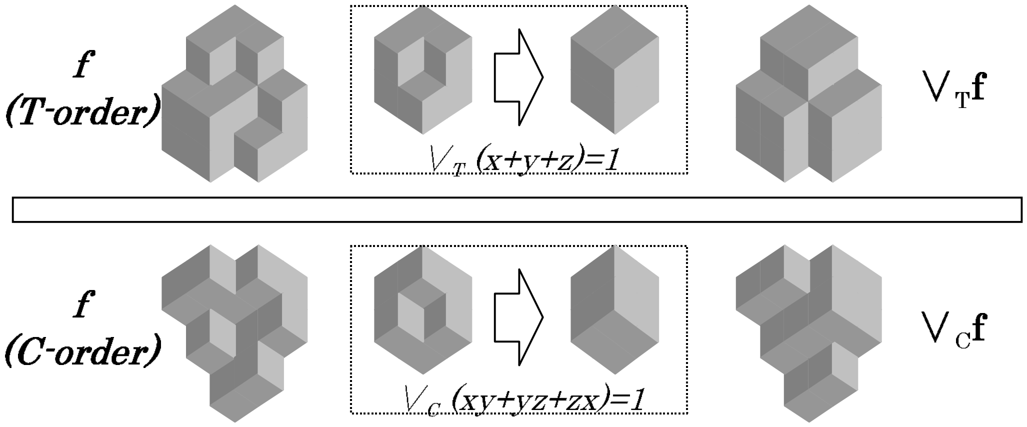
$\Leftrightarrow l_i \leq m_i \leq (0 \quad i < N)$

Tangent-order $\geq_T$ and Cotangent-order $\geq_C$

*$e = x_0 x_1 \ldots x_{N-1}$

5.2 Join operator $\vee$

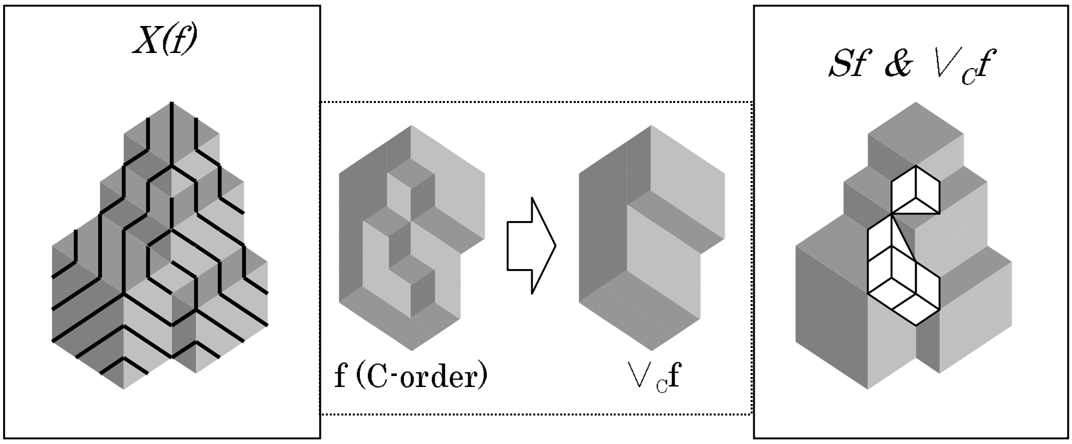
(2) Join operator $\vee$



Cover of f $\in$ PL $\Leftrightarrow$ $\vee_C \mathbf{f}$

5.3 Decomposition of B

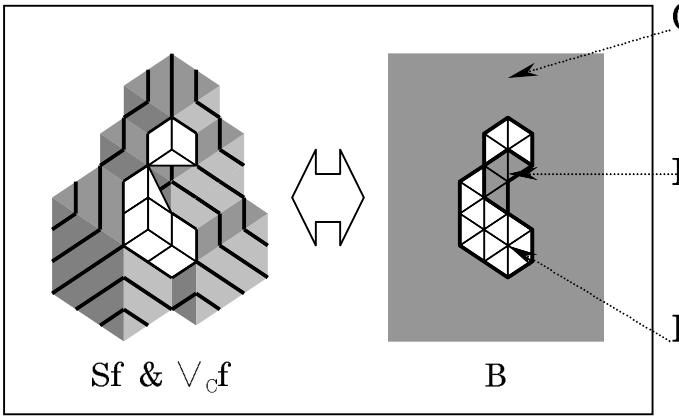
(1) Sf (f ∈ PL) and $\vee_C f$



Polynomial section and its cover

5.3 Decomposition of B

(2) Decomposition of B along X(f)



Out(f)

⇔ Tiles over $\vee_C f$

Bd(f)

⇔ Tiles intersec. $\vee_C f$

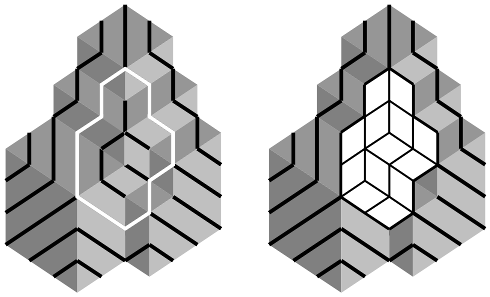
In(f)

⇔ Tiles under $\vee_C f$

$$\mathbf{B = In(f) + Bd(f) + Out(f)}$$

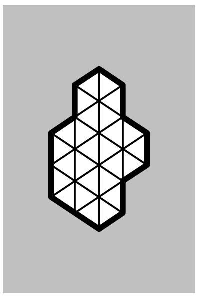
5.4 Definition

(1) Definition of prehetero numbers



X(f) Sf & $\vee_C$f

⇔

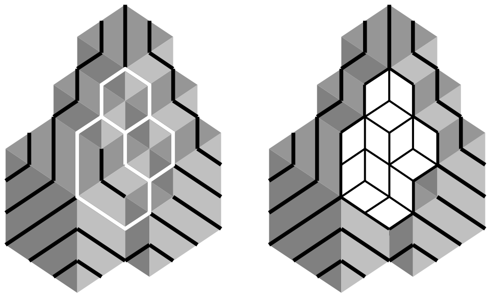


In(f)
(Bd(f) = ∅)

f ∈ PHN^{N-1} ⇔ f ∈ PL & Bd(f) = ∅

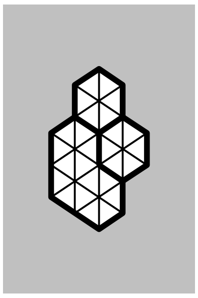
5.4 Definition

(2) Decomposition of f ∈ PHN into closed orbits



X(f) Sf & $\vee_C$f

⇔



ΣInV(f(f) = $_i$)

$$f = f_0 \oplus f_1 \oplus \dots \oplus f_{m-1} \quad (f_i \in AHN)$$

PHN^{N-1} ⊂ AHN^{N-1} (PHN^2 = AHN^2)

5.5 Algebra of $\mathbb{PHN}$

(1) Addition of $p_0, p_1 \in \mathbb{PHN}$

$$\boxed{f_0 \oplus f_1 := f \quad \text{if} \quad \mathrm{In}(f) = \mathrm{In}(f_0)+\mathrm{In}(f_1)}$$

Prime factoring of $f \in \mathbb{PHN}$

(a) In $\mathbb{PHN}$: $f = p_0 \oplus p_1 \oplus \ldots \oplus p_{m-1}$

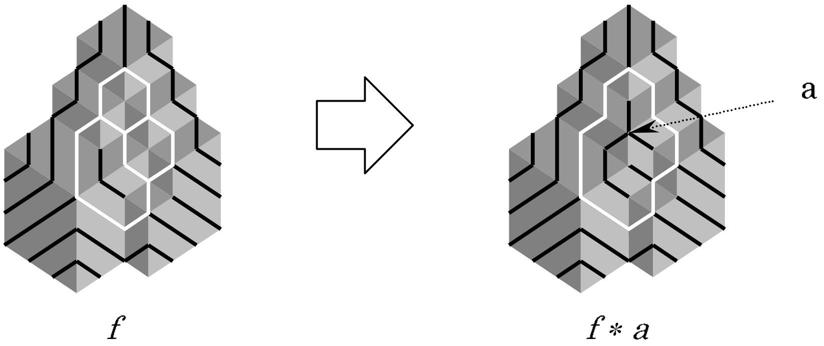
(b) In $\mathbb{AHN}$: $p_i = lp_{i,0} \oplus lp_{i,1} \oplus \ldots \oplus lp_{i,k(i)-1}$ $(0 \le i < m)$

$$\mathrm{In}(p_0 \oplus p_1) = \mathrm{In}(p_0) + \mathrm{In}(p_1)$$

5.5 Algebra of $\mathbb{PHN}$

(2) Action of $a \in PL$ on $f \in \mathbb{PHN}$

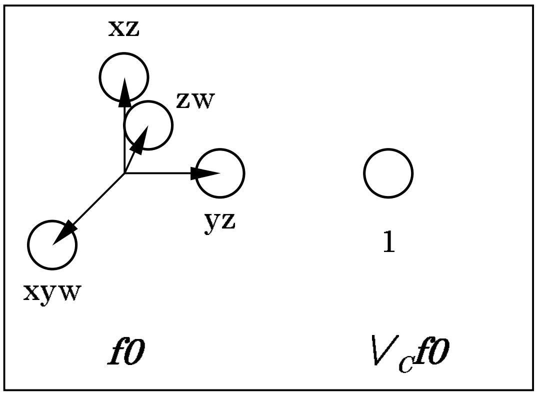
$$\boxed{f * a := f + a \quad \text{if} \quad f + a \in \mathbb{PHN}}$$

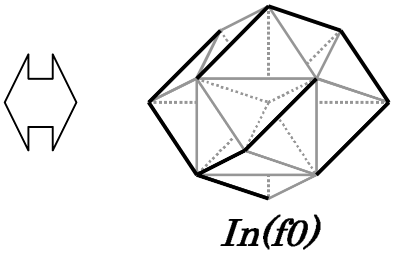


$$* : \mathbb{PHN} \times \mathbf{PL} \to \mathbb{PHN}$$

5.6 Example (N=4)

(1) Rhombic dodecahedron



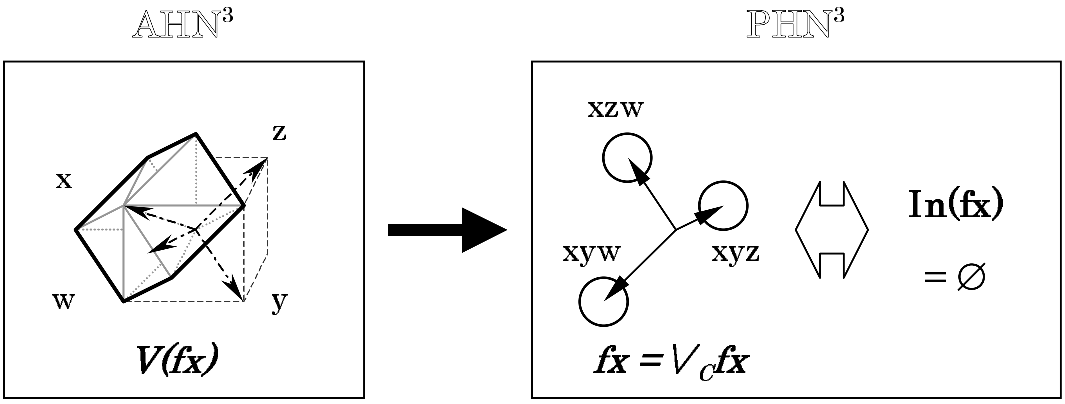


U-D-D-U-D-U-U-D
-U-D-D-U-D-U-U-D
-U-D-D-U-D-U-U-D

$$\mathbf{f0 = xyw + xz + yz + zw \in PHN^3}$$

5.6 Example (N=4)

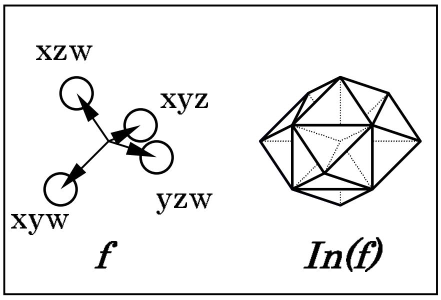
(2) $PHN^3 \subset AHN^3$



$$\mathbf{fx = xyz + xyw + xzw \in AHN^3 - PHN^3}$$

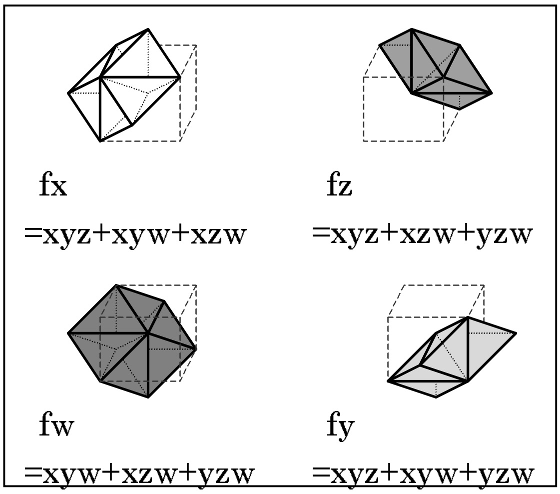
5.7 Example (N=4)

(1) Loop decomposition of rhombic dodecahedron



$f = xyz + xyw + xzw + yzw \in \mathbb{PHN}^3$

=



fx =xyz+xyw+xzw

fz =xyz+xzw+yzw

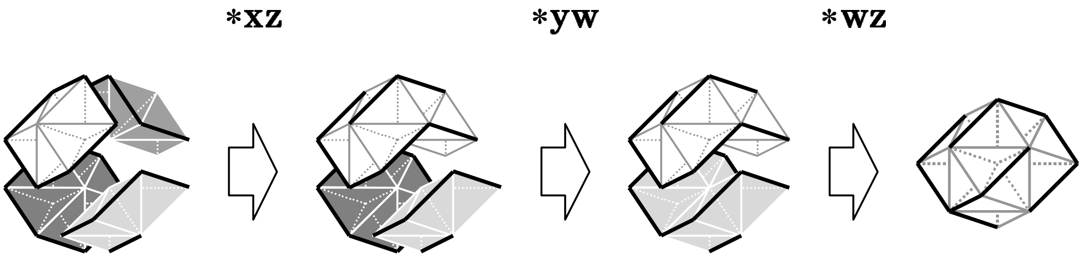
fw =xyw+xzw+yzw

fy =xyz+xyw+yzw

$$\boxed{\mathbf{f = fx \oplus fy \oplus fz \oplus fw}}$$

5.7 Example (N=4)

(2) Action of PL on $\mathbb{PHN}$



$f = fx \oplus fy \oplus fz \oplus fw$

$f{*}xz = fxz \oplus fy \oplus fw$

$f{*}\,(xz+yw) = fxz \oplus fyw$

$f{*}\,(xz+yz+wz) = f0$

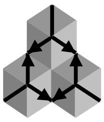
$$\boxed{\mathbf{f * (xz+yz+wz) \quad = \quad f0}}$$

6.1 D^2-moduli

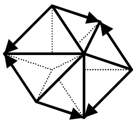
(1) Moduli space of affine loops

ALP (primes of AHN)

obt(f)={ t[i] }



xy+yz+zx



xyw+xzw+yzw

D^2-code

(2-dim.)

(3-dim.)

M

cd(f) := { $D^2f(t[i])$ }

U-D-U-D-U-D

U-D- …… -U

U-U- …… -D

……

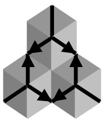
M := the set of all finite sequences of { U, D }

6.1 D^2-moduli

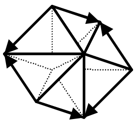
(2) Implementation of a D^2-code

Orbits in B

$V_m(cd, t_0)$



xy+yz+zx



xyw+xzw+yzw

Imple.

(2-dim.)

(3-dim.)

M

cd={ c[i] }

U-D-U-D-U-D

U-D- …… -U

U-U- …… -D

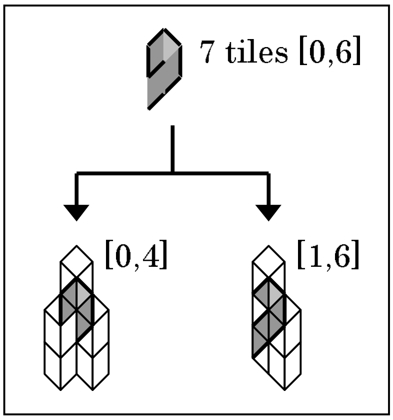
……

$V_m(\text{ cd, } t_0) = \{ t[i] : t[0]=t_0 \ \& \ D^2(t[i]) = c[i] \}$

6.2 Polynomial representation

(1) Locally affine orbit

Orbits in B^2



M

D-U-D-U-D-D-D

2-dim. Imple. ←

→

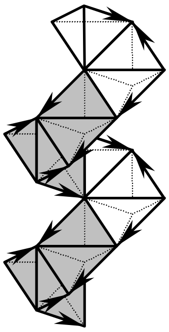
Polynomial representation
(Hetero scheme)

$xy + yz + zx$ in [0,4]
$xy + xz$ in [1,6]

Patchwork of affine orbits – "*Manifold*" –

6.2 Polynomial representation

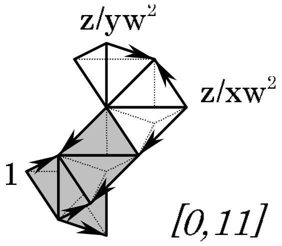
(2) Example

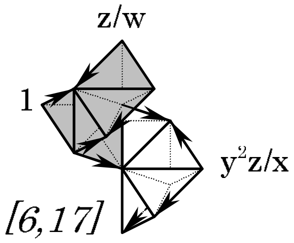


D-D-U-U-D-D-D-D-U-U-D-D
-D-D-U-U-D-D-D-D-U-U-D-D
-D-D-U-U-D-D-D-D-U-U-D-D

⇩

$z/yw^2 + z/xw^2 + 1$ in [0,11]
$z/w + 1 + y^2z/x$ in [6,17]
$yz + y^2z/x + y^2w^2$ in [12,23]

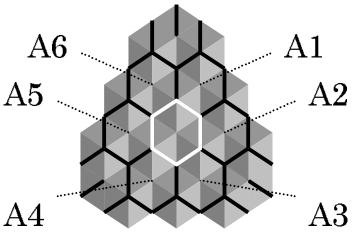




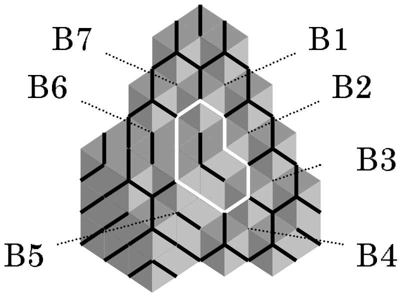
Polynomial representation of *helix (DNA)*

6.3 Galois theory of ``proteins”(1)

(1) Petals



$f_A = xy + xz + yz$
and its petals

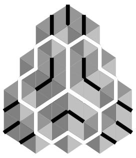


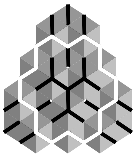
$f_B = x^2y^2z + x^2z^3 + yz^2$
and its petals

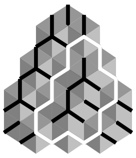
Petals := Surrounding loops

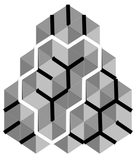
6.3 Galois theory of ``proteins”(1)

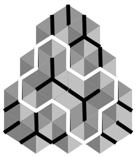
(2) Equations among petals of f_A

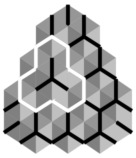

(a)

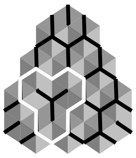

(b) A0

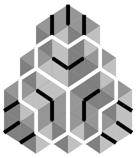

(c)


(d)


(e)


(f)

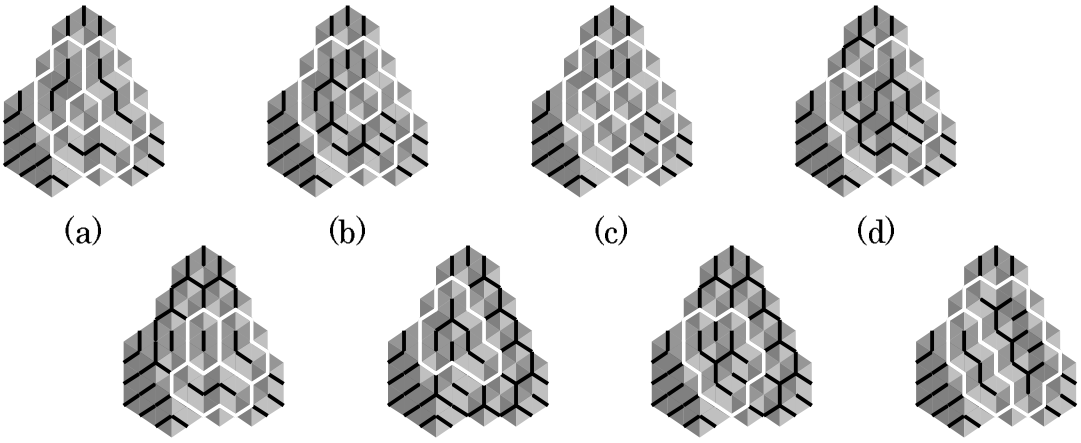

(g)


(h)

$$\left(f_A \oplus p(A1) \oplus p(A2) \oplus \ldots \oplus p(A6) \right) * \left(x + y + z \right) = p(A0)$$

6.4 Galois theory of ``proteins”(2)

(1) Equations among petals of f_B



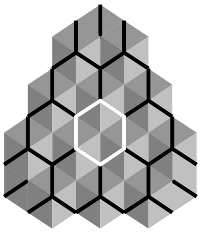
(a) (b) (c) (d)

(e) (f) (g) B8 (h)

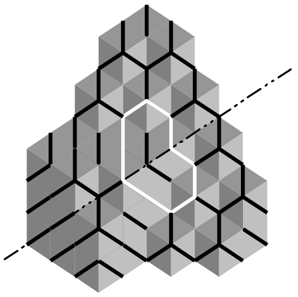
$$(f_B \oplus p(B5) \oplus p(B6)) * (-yz^2 + xyz^2) = p(B8)$$

6.4 Galois theory of ``proteins”(2)

(2) Symmetry of an affine hetero number



f_A ⇔ Point symmetry



f_B ⇔ Line symmetry

Galois group of $\mathbb{AHN}$ ⇔ Permutations of petals

7. Summary

(1) Features of hetero number system

<Biology and hetero number theory>

- Gene ⇔ D^2[protein]
- Protein folding ⇔ Integration
- Protein-protein interaction ⇔ Addition

<Natural numbers and hetero number theory>

- Higher dimensional extension of $\mathbb{N}$ w.r.t. addition
- Genetic coding of $\mathbb{N}$ (2nd derivative of $\mathbb{N}$)

Biology and Mathematics

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7. Summary

(2) Several topics

- D^2-moduli
- Implementation of D^2-code
- Galois theory of $\mathbb{AHN}$

- Relation of heterology (biology) and homology (physics)

… physics as a ``dual" of biology

Moduli, representation & Galois theory